Net Present Value Optimization for Multi Project Capital Constrained Scheduling Problem

S. H. Yakhchali

Abstract**—**One of the important things about project management particularly for project manager role is scheduling. Scheduling of a project can be done for different purposes and with varying scales of details included depending on project's nature, requirements, limitations etc. generally organizations' goals and targets of planning for a project can be translated to scheduling models such as RCPSP that is well known for being used in many researches and even in real cases.

I. Introduction

One of the important things about project management particularly for project manager role is scheduling. Scheduling of a project can be done for different purposes and with varying scales of details included depending on project's nature, requirements, limitations etc. generally organizations' goals and targets of planning for a project can be translated to scheduling models such as RCPSP that is well known for being used in many researches and even in real cases.

The main concept of RCPSP is that how we should do the project activities in order to minimize the overall project duration. This happens under the situation that resources (renewables, non-renewables or even both) are constrained and this limitation affects activities scheduling plan and finally project's overall duration.

Beside minimizing project completion time there are other scenarios that can be used under specific circumstances and other interests. minimizing project total costs, maximizing project quality and maximizing project net present value (that is this papers objective) regularly are main goals of planning a project.

In order to discuss further about this papers innovation, first we introduce NPV (net present value) in brief. NPV refers to a situation that your money in next period (usually a year) will not have the same value that it has now. For example if you have 10 dollars now it is more precious than same 10 dollars in some years later. This happens for a lot of reasons like inflation, loan interest rates, macroeconomic variables and so forth that are not in the scope of this topic.

There are many papers and researches related to optimizing NPV for a project that has constrained resources and financial limitations (cash flow and budget limitations, capital constraint etc.) this problem comes into the mind of the project manager when he/she should know how the project should be executed to meet budget limitations and limited resources availabilities.there are a lot of good researches and solutions for this problem that can be accessed in related journals.

But what thing that drove us to this topic was that what if there are a few related projects, lets say a program, and they use a shared cost center in organization. the manager should answer to this question that how is he going to handle budget and resources limitations in order to make the organization more profitable, cost saving and optimized.

This paper seeks an optimized solution for multi project scheduling problem with capital constraints. The final goal is maximizing NPV for all related projects combined. No need to mention that it is possible that the total solution may not be the best one for each project alone, but one thing that is important is that in aggregate we reach optimal or near optimal solution that maximizes program's net present value.

II. Literature review

As this paper's main focus is on scheduling and planning a program (especially applicable for a program manager) in order to maximize the net present value for all of the projects, combined; it is worthwhile to put an effort on identifying what researches have been done about this topic. We start with single project NPV optimization and some of it's extensions and modifications. Next subtitle will be multi project (we address them as programs in this paper) scheduling problem, methods and innovations. Then they will be put on a summary and a comparison between each one's elements will be made. Then The next step will be about contribution.

A. net present value optimization problems

The story of net present value optimisation in networks begins with russel's " cash Flows In networks" back in 1970. It was for first time that instead of minimizing project's total duration he et al used linear programing for maximizing project net present value due to cash flows.

Since then areas that have been covered in NPV optimization concepts are as follows:

● With considering single project scheduling and single mode activities Vanhoucke et al. (2003) studied unconstrained project scheduling problem with objective of maximizing net present value of project cash flows.

● Pieter Leyman, Mario Vanhoucke (2016) studied resource and capital constrained project scheduling problem with discounted cash flows known as RCCPSPDC for a single project with considering three payment methods.

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●A single and multi mode project scheduling with constrained resources introduced by Pieter Leyman and Mario Vanhoucke (2015).

There are many other researches and studies that if we want to summarize them in the way that we need, table below would be a good explanation:

Authors (Year)	Resource constraints	Capital constraints	Other extensions
Vanhoucke et al. (2003)			
Hosseini et al. (2014)	x		
Chen and Zhang (2012)	x		Dead line penalty/bonous
Aboutalebi et al. (2012)	x		Multi objective scheduling
Kazemi and Tavakkoli-Moghaddam (2010)	x		Multi objective and multi mode
Leyman, P., Vanhoucke, M (2015)	x		Multi mode
Ulusoy and Cebelli (2000)	x		
Kavlak et al. (2009)	x		Client-contractor trade off
He et al. (2012)		x	Multi mode
Leyman, P., Vanhoucke, M (2016)	x	x	
This work		x	Multi project+deadline penalty+cost- time trade off

Fig. 1. NPV optimization literature review

B. multi project scheduling problem

Managing multi-projects or programs is important mainly for organization's senior manager as they want to know how to spend their resources and budget in order to get maximum benefit from a program.

We understood that managing multi project with the goal of NPV optimization is a topic that is neglected. We can see resource constrained multi project scheduling problem in IIM Rohtak, Rohtak (2014)'s paper or multi-mode and resource constrained multi-project scheduling in Wauters, et al (2013)'s work. But the goal function is minimizing project make span and there is no such a thing as capital constraints.

Figure 2 describes the works that are done in this topic.

Authors (Year)	Goal function	Resource constraints	Other extensions
IIM Rohtak,Rohtak (2014)	timing	x	Priority rules
Wauters, et al (2013)	timing	x	Multi mode
Umut Beşikci, et al (2015)	timing	x	Multi mode, portfolio resource problem
Doreen Krüger, Armin Scholl (2009)	timing	x	sequence-dependent transfer times
Tyson R. Browning, Ali	timing	x	Project and portfolio

A. Yassine (2010)			lateness
J.F. Gonc¸alves et al (2008)	timing	x	
This work	NPV optimization		Capital constraint+deadline penalty+cost-time trade off

Fig. 2. multi project scheduling literature review

It is obvious that such a crucial subject as maximizing program's net present value is like a puzzle's missing piece in project scheduling problem context.

In next sessions we have problem definition and modeling (session 3), solution,running computational codes and experiment (session 4), conclusion (session 5) and acknowledgement (session 6).

III. Problem Definition and Modelling

As discussed above the problem is to maximize a program's net present value when there is limitation for capital that is budgeted for all of the projects combined. So we have number of projects that are going to be scheduled and managed simultaneously by program manager. As level of management goes higher, we can expect less details about activities and due to PMI one of key responsibilities of a program manager is reporting to top manager about work progress, cost control and so forth. By the way we can see projects with their major phases and activities to implement NPV optimization for them, not detailed activities. And capital constraints also apply for project phases instead of activities as concept of managing costs and budget control is not in activity level but a group of related activities also known as work packages. If we want to give a clear picture of the problem it's mainly about scheduling major phases of the project in order to meet budget requirements and maximize the entire program's net present value.

A. Research scope and assumptions

To clear out we what are going to deal with, it is necessary to address scope of work, boundaries and assumptions. In this topic we have numbers of projects that each one one of them has some activities (conceptually we consider them as phases or work packages due to program management theories and context) with dependency relationships between them. Actually our intention of using 0the words 'program' and 'project phases' in practice is 'multi project' and 'activities' respectively.

Just like in real world projects, programs or any other work, we have an initial capital that is assigned to entire program. This initial investment is not enough for completion of the projects but to start them, making progress and receive payments from the employer. This assumption implies that beside considering an initial investment and managing it in an efficient way, cash flows are also included.it is assumed that payments occur at end of the Activities as they are major phases of the projects. But cost expenditures will be accrued over the activities duration.

An example of how cash flows occur will be as follows:

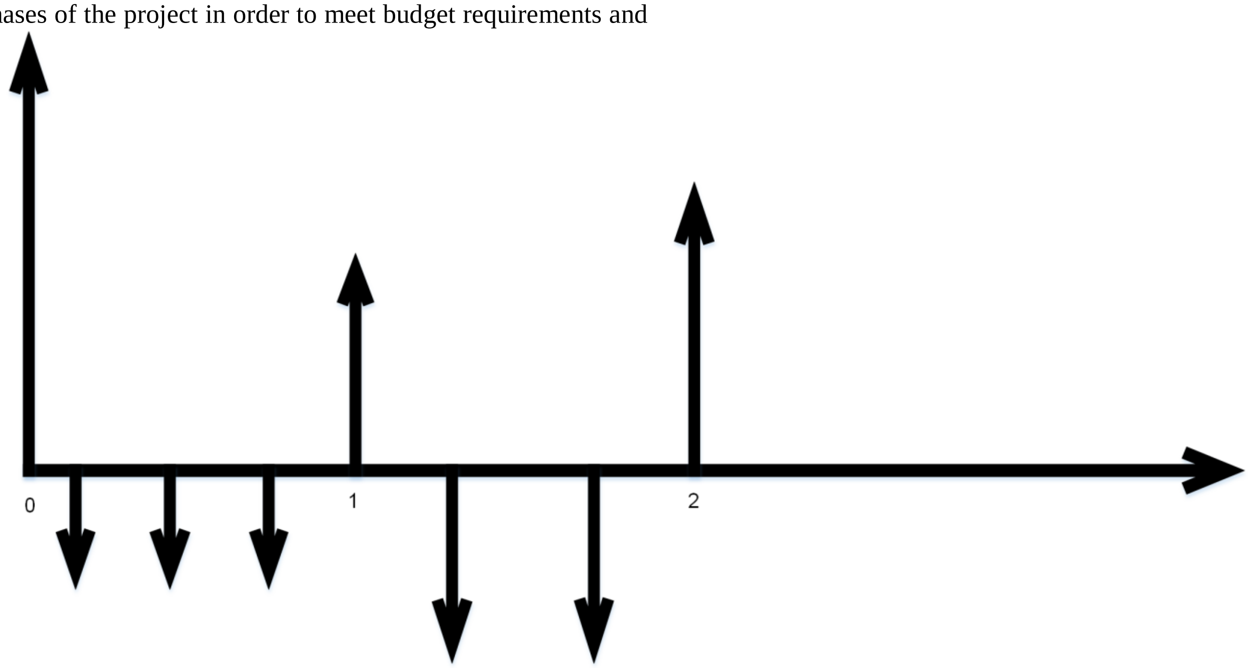

Fig 3. Example of projects' cash flows

Another important thing about cash flows is that in the end of each period capital must be positive. In other words negative cash flow is not allowed.Soft scheduling of the project is considered as delay is allowed but with a penalty cost that will be added to the goal function.

In calculating project's net present value discount rate is considered continues.

Below is the list of abbreviations and nominations that have been used in next sessions.

i	Projects set (i=1,2,3,...,n)
j	Activities set (j=0,1,2,3,...,m+1)
V	Precedence dependencies matrix
S(t)	set of activities which has started before or in time t
F(t)	set of activities which has completed before or in time t
s_{ij}	Start time of activity j that belongs to project i (variable)
d_{ij}	duration of activity j that belongs to project i (parameter)
f_{ij}	Finish time of the activity j in project i ($=s_{ij}+d_{ij}$)
γ_{ij}	Duration % completion of the activity j in project i ($=(t-s_{ij})/d_{ij}$ (variable)
$c_{ij,in}$	Cash inflow of activity j that belongs to project i (parameter)
$c_{ij,out}$	Cash outflow of activity j that belongs to project i (parameter)
r	Annual discount rate (parameter)
α_i	Penalty/bonus rate for tardiness/earliness of project i (parameter)
pf_i	Deadline for project i(parameter)
t	Time period = 0,1,2,...

B. Optimization model

1. goal function

To meet the assumptions, boundaries and requirements of the research that have been discussed in previous sections and to maximize total net present value of the projects; the goal function could be as below:

$$Maximize\ NPV = \sum_i \sum_j \left(c_{ij,in} \times e^{-r.f_{ij}} + c_{ij,out} \times e^{-r.\frac{f_{ij}+s_{ij}}{2}} \right) (1) + \sum_i \alpha_i T_i \times e^{-r.f_{ij+1}}\ (2)$$

$$T_i = min\,(0,\ pf_i - f_{ij+1})$$

For sorting out and giving better explanation we seperated part (1) from part (2) of the goal function:

(1):main part of the goal function that is equal to net present value of all activities in all projects. As inflow of a project occurs at activities finish times for calculating npv f_{ij} is considered for discount period. In the other hand due to assumptions, projects' outflows distributed on activities duration. If we consider it linear, we can skip calculations through integral and just use a simple average of activities' start and finish dates. So the formula above is taking back inflows with f_{ij} and outflows with $(s_{ij}+f_{ij})/2$.

(2): this part implies penalty for missing each project's deadline and also a bonus for completing projects before deadline. α_i is the rate of penalty/bonus that will be applied if.f_{ij+1} is a dummy variable for project i finish date. This formula adds penalty/bonus to the projects' overall NPV.

2. Constraints

(1) $s_{ij} + d_{ij} = f_{ij} \quad ; \quad i = 1,2,\dots,n\ , j = 0,1,2,\dots,m+1$

This constraint describes the relation between activities start and finish time.

(2) $s_{ij} \ \le \ f_{ik} \quad ; \quad i = 1,2,\dots,n\ , j,k \in V$

Task dependencies are shown as simple FS relationships inside each projects. It is considerable that due to managing multi projects in a high level management process, lag or lead concept is replaced for penalty/bonus for entire project that will be shown in next constraints.

(3) $c_0 + \sum_{i=1}^{n} \sum_{j \in F(t)} c_{ij,in} + \sum_{i=1}^{n} \sum_{j \in S(t)} c_{ij,out} \times \gamma_{ij}$; $t = 0,1,2,\dots,max(f_{im+1})$, $i = 1,2,\dots,n$

$$\gamma_{ij} = (t - s_{ij})/d_{ij}$$

This constraint prevents capital to be negative at any time. c0 illustrates the initial budget that is assigned to projects. As payments occur at activities completion, second expression sums up all the inflows that has been paid until time t. Second sigma implies that only payments that have been occurred before time t are considered. This payments are connected to activities that are completed before this time.

Same logic applies for third expression. It means that as out flows are connected to activities that are already started, set of activities that their start times are before time t must be considered. And finally because of problem's assumption which outflows are accrued over activities duration, activities' percent complete is multiplied to the expression.

(4) s_{ij} , $f_{ij} \in int^{+}$; $\gamma_{ij} \in Q^{+}$;.

This constraint is just a simple variable definition.

This model at the end will give each activities start time in order to maximize entire program's NPV with considering penalty/bonus for missing deadlines or completing projects before their deadlines.

The whole model in one piece will be as below:

$$Maximize\ NPV = \sum_i \sum_j \left(c_{ij,in} \times e^{-r.f_{ij}} + c_{ij,out} \times e^{-r.\frac{f_{ij}+s_{ij}}{2}} \right) (1) + \sum_i \alpha_i T_i \times e^{-r.f_{ij+1}}\ (2)$$

Subjected to:

$$T_i = min\,(0,\, pf_i - f_{ij+1})$$
$$s_{ij} + d_{ij} = f_{ij} \quad ; \quad i = 1,2,\dots,n\,, j = 0,1,2,\dots,m+1$$
$$s_{ij} \le f_{ik} \quad ; \quad i = 1,2,\dots,n\,, j,k \in V$$
$$c_0 + \sum_{i=1}^{n} \sum_{j \in F(t)} c_{ij,in} + \sum_{i=1}^{n} \sum_{j \in S(t)} c_{ij,out} \times \gamma_{ij}\ ; \quad t = 0,1,2,\dots,max(f_{im+1})\,, \ i = 1,2,\dots,n$$
$$\gamma_{ij} = (t - s_{ij})/d_{ij}$$
$$s_{ij}\,,\, f_{ij} \in int^{+};\ \gamma_{ij} \in Q^{+};$$

C. solution

To provide a solution for the model a genetic algorithm (GA) is proposed with specialized

local search. Three parts of the solution would be as follows:

- Initial schedule: to create initial population for the genetic algorithm we use serial schedule generation scheme of Kolisch (1996). Priority list can be obtained from GA.this will also ensures that there is at least one feasible solution to the problem due to deadline constraint. That is because to find out if there is not any other constraint, meeting deadline is possible by itself.
- Meet capital constraint and making improvement: in order to make sure that capital is not negative at any time period from t=0 to entire program's finish time, some activities must be delayed. Some can be delayed without making any interruption in project's deadline with just using it's float with their feasible range. But clearly activities in critical path can cause tardiness for entire project. But meeting this constraint is Inevitable because having negative constraint due to our assumption isnot allowed and that is because we assume that it is not possible to borrow money, postpone payments and so forth. So in a trade off between meeting deadline and having positive capital, capital takes the upper hand.
- Optimizing NPV: after finding feasible solutions to meet capital constraint, now it's time to go after optimizing the results by delaying some activities in order to achieve more net present value for the project and consequently entire program. This part is only possible when a capital feasible solution is already in hand.

Figure 1 below gives an overview of the flowchart for the solution:

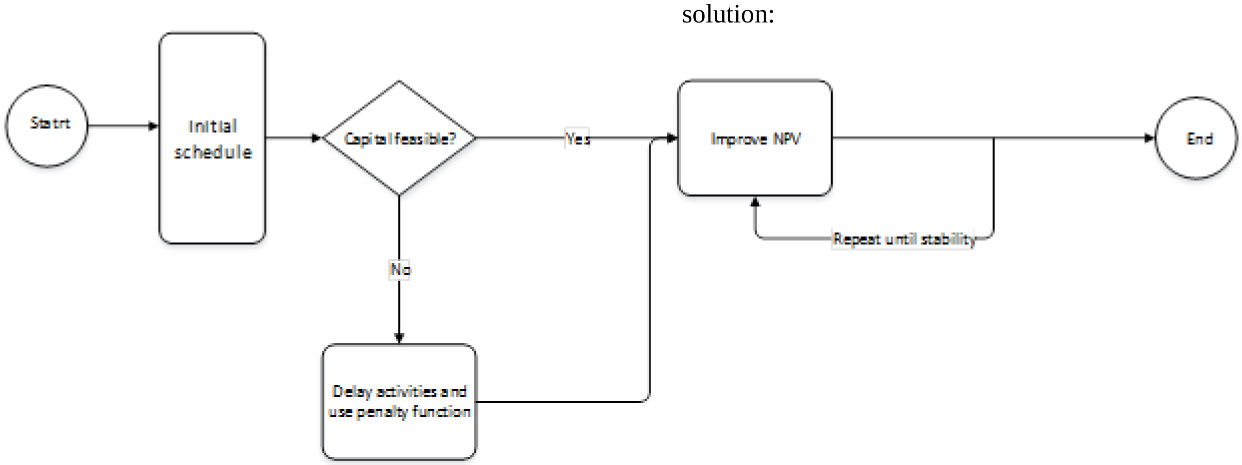

Fig. 4. problem solution flowchart

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D. Optimization model

The evolutionary algorithms such as genetic algorithm are among the most applicable and practical optimization techniques. In spite of exact methods that are so sensitive to non-linear functions or constraints, metaheuristics are kind of non-linear proof that we can see clearly in optimization context that many complex and non-linear models are being solved with this

method.

As this papers model is a more complex and advanced version of simple NPV optimization

models, and with considering the goal function and the constraints it is recommended to use a

metaheuristic method to find an optimal or near optimal (time feasible) solution to the problem.

Building up chromosomes For each project (i) the matrix below represents the chromosome that it's solution would be the main problem's solution:

j \ t	1	2	3	4	5	6	…	$Sum_j(D_{ij})$
1	**1**	**1**	**1**	**1**	**-**	**-**	**-**	**4**
2	**-**	**-**	**-**	**1**	**1**	**-**	**-**	**2**
3	**-**	**1**	**1**	**1**	**1**	**-**	**-**	**4**
4	**-**	**-**	**-**	**-**	**-**	**1**	**…**	**…**
…	**…**	**…**	**…**	**…**	**…**	**…**	**…**	**…**
m	**-**	**-**	**-**	**1**	**1**	**1**	**…**	**…**

Fig. 5. Chromosom matrix sample

In the first column, j is the activity number related to project I;

In the first row, t is the time;

In the binary matrix, in each row first "1" is activity's start time (s_{ij}) and the last "1" represents the activity's finish time (f_{ij}) so the number of "1" s in each row will give the duration (d_{ij}).

For example activity number 3 starts at t = 2 and will be finished at t = 6 and the duration is 5.

Network feasibility

For each project, first it has to be considered that dependency constraints are met and the

network is feasible by itself due to dead line. For this purpose with considering dependency

matrix that is a input to the problem (set V); finish time of the activity j (f_{ij}) must be before start time of the activity k (s_{ik}). In other words the last "1" in row j must be before than the first "1" in row k. if this requirement is not met then the (s_{ik}) or the first "1" will be delayed.

Capital feasibility

To ensure that capital will remain positive at any time, at any time t, the model should determine each activities status and calculate in/outcomes and finally update total capital. For this purpose if an activity has completed till time t, it's cash in will be added to the capital. Otherwise only cash out will be subtracted from total capital with respect to activity's percent complete.

Metaheuristic algorithm

Due to the discussions above, the designed pseudo code for the problem will be as follows:

- Read input parameters
- Set algorithm parameters
- Make initial population
    - Assess the initial population (solution)
- Run the loop below till stoping condition meets:
    - Select the best chromosomes (x)
    - For other chromosomes (y) run the loop below:
        - If the cross over operator is selected, substitute the chromosome y with the child of parents x and y
        - If the mutation operator is selected change j with it's mutated form
    - Check the feasibility
    - Evaluate the population and sort them
- Illustrate and save the results

Fig. 6. GA algoritm psuedo code

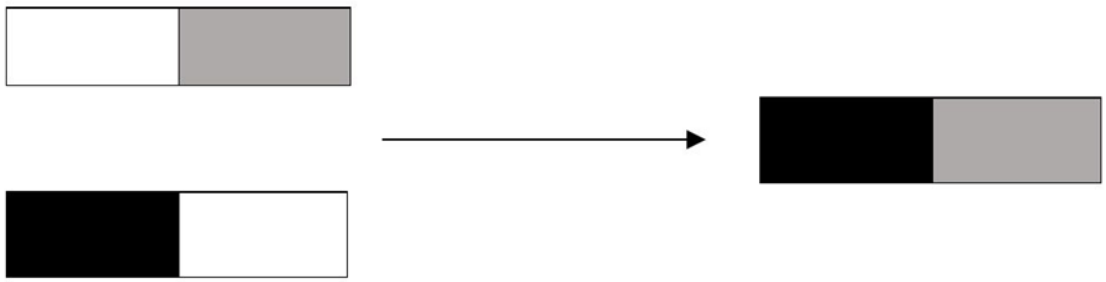
E. cross over operator

the process below will be used for crossing chromosomes:

1) calculate break-point based on chromosomes dimensions through formula below: Break-point = round () Sum j (D_{ij}) /2)

2) make children with considering break-point above

3) make a new chromosome from first half of second chromosome and second half of first chromosome.



F. Stop condition

Running algorithm in the computer will go on until either a specific and rational time or the solution (chromosome) after each iteration represents no change or less than a specific amount.

For bring this concept to mathematics world formula below illustrates the main idea:

$$\alpha_1 \times \left|\frac{mean_{t-1} - mean_t}{mean_{t-1}}\right| + \alpha_2 \times \left|\frac{max_{t-1} - max_t}{max_{t-1}}\right| \leq \varepsilon$$

α_1, α_2 and ε are parameters. For this model they are equal to 0.6, 0.4 and 0.025 respectively.

〖 mean〗 _t equals to mean of objective function in time t.

〖 max〗 _t equals to maximum value of objective function in time t.

This formula ensures that if the inequality exists the metaheuristic algorithm will continue running regardless of the time.

References

[1] Pieter Leyman, Mario Vanhoucke, CapitalĐ and resourceĐconstrained projectscheduling with net present value optimization, European Journal of Operational Research (2016)

[2] Russell, A. (1970). Cash flows in networks. Management Science, 16:357–373. https://doi.org/10.1287/mnsc.16.5.357

[3] Vanhoucke, M., Demeulemeester, E., and Herroelen, W. (2003). Progress payments in project scheduling problems.European Journal of Operational Research, 148:604–620. https://doi.org/10.1016/S0377-2217(02)00452-6

[4] Leyman, P., Vanhoucke, M., Payment models and net present value optimization for resource–constrained project scheduling, Computers & Industrial Engineering (2015) https://doi.org/10.1016/j.cie.2015.11.008

[5] Hosseini, Z.S., Pour, J.H., Roghanian, E., 2014. A bi–objective pre–emption multi-mode resource constrained project scheduling problem with due dates in the activities. Journal of Optimization in Industrial Engineering 15, 15–25.

[6] Chen, W.N., Zhang, J., 2012. Scheduling multi-mode projects under uncertainty to optimize cash flows: A monte carlo ant colony system approach. Journal of Computer Science and Technology 27, 950–965 https://doi.org/10.1007/s11390-012-1276-2

[7] Aboutalebi, R., Najafi, A., Ghorashi, B., 2012. Solving multi-mode resource-constrained project scheduling problem using twomulti objective evolutionary algorithms. African Journal of Business Management 6, 4057–4065. https://doi.org/10.5897/AJBM11.1628

[8] Kazemi, F., Tavakkoli-Moghaddam, R., 2010. Solving a multi–objective multi–mode resource–constrained project scheduling problem with discounted cash flows. 6th International Project Management Conference

[9] He, Z., Liu, R., and Jia, T. (2012). Metaheuristics for multi-mode capital-constrained project payment scheduling. European Journal of Operational Research, 223:605–613. https://doi.org/10.1016/j.ejor.2012.07.014

[10] Ulusoy, G., Cebelli, S., 2000. An equitable approach to the payment scheduling problem in project management. European Journal of Operational Research 127, 262–278. https://doi.org/10.1016/S0377-2217(99)00499-3

[11] Kavlak, N., Ulusoy, G., Sivrikaya-S¸erifo˘glu, F., Birbil, I., 2009. Client-contractor bargaining on net present value in project scheduling with limited resources. Naval Research Logistics 56, 93–112 https://doi.org/10.1002/nav.20331

[12] IIM Rohtak, Rohtak , 2014 . Resource Constrained Multi-Project Scheduling with Priority Rules & Analytic Hierarchy Process. Procedia Engineering 69 (2014) 725 – 734 https://doi.org/10.1016/j.proeng.2014.03.048

[13] Wauters, T., Kinable, J., Smet, P. et al. J Sched (2016) 19: 271. https://doi.org/10.1007/s10951-014-0402-0

[14] Beşikci, U., Bilge, Ü., Ulusoy, G., (2014). Multi-Mode Resource Constrained Multi-Project Scheduling and Resource Portfolio Problem, European Journal of Operational Research (2014), doi: http://dx.doi.org/10.1016/j.ejor.2014.06.025

[15] Doreen Krüger, Armin Scholl, (2009). A heuristic solution framework for the resource constrained (multi-)project scheduling problem with sequence-dependent transfer times, European Journal of Operational Research 197 (2009) 492–508. https://doi.org/10.1016/j.ejor.2008.07.036

[16] J.F. Gonc¸alves , J.J.M. Mendes , M.G.C. Resende, 2008. A genetic algorithm for the resource constrained multi-project scheduling problem. European Journal of Operational Research 189 (2008) 1171–1190 https://doi.org/10.1016/j.ejor.2006.06.074