

Complex Analyses Of Shear Lag Problem To Mitigate Earthquake Damages

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Abstract—The structural problems are one of the main focus of study nowadays due to the recent events of Turkey, despite the significant episodes around the world. For this structural elements study it is common to have approximations. Involving plates connected by connectors to metal beams. Shear lag effect is an unexpected major phenomenon that controls the design of tall buildings using framed tube system. The intention is to use models that are not very complex and easy to use, which, on the one hand, guarantee the structure a good behavior in service and, on the other hand, allow the designer an easy reading of the model results in such a way that all the details of the structure are made clear and simple. The existence of a diversity of models for stairwells derives from the fact that the common bar elements do not consider the difference between the shear center and the center of mass and do not model the effects of shear-lag and warping. To compare the different models, in a first phase, extensive parametric studies of the response of isolated boxes to horizontal static actions will be carried out. The main parameters to be varied will be: applied action, cross-sectional shape, box height, discretization in plan and height and the type of modelling. The results of the different models will also be compared with the solution given by the theory of linear parts, which does not include the shear-lag effect, but considers warping due to torsion. In a second phase, the analysis of building structures will be carried out using the models studied in the first phase. It is hoped that these studies will be able to guide designers on the advantages and disadvantages of different types of modeling, including the type of associated errors and the respective field of application. Comparing displacements. The objective of the present paper is to present the calculation structure using shear lag. Shear lag is a concept used to explain uneven tension distribution in connected elements. Further conclusions can be made for renewable energy systems decentralization on the top of the buildings.

Index Terms— Shear Lag, Effects, Steel and Concrete Tension, Tube, Concrete Structural Systems, Tall Buildings.

I. INTRODUCTION

Occurrence of shear lag in buildings is reported in the literature for several years ago and all over the world [1-8], but explanation of its origin and comprehensive studies of it are lacking. Historically, the problem of Shear Lag effects was first approached by Von Karmen and successively by several

authors. Building suffers from shear lag effects which cause a nonlinear distribution of axial stresses along the face of the building. Major advancements in structural engineering have been the development of different structural systems that allow for higher buildings. As the height of the building increases, the lateral resisting system becomes more important than the structural system that resists the gravitational loads. Also, introducing renewable energy systems at the top of building cause an extra height (such as solar systems) and vibrations (such as wind power) that affect the structure equilibrium that was firstly designed.

The need to consider the effects of Shear Lag in the design of structures initially arose in the field of aeronautical engineering [9-14] but immediately naval architects [15-17] and civil engineers [18-19] were faced with the need to address the same problem.

However, only in 1990 a methodical study [20] allowed the identification of parameters that governs the effects of the phenomenon on the behaviour of wide-beam beams. The results of the studies were used as the basis for the wide chord design rules included in the most recent international legislation.

II. DISCRETIZATION OF STRUCTURAL MODELS

For the present study, we show results of 3 types of models we use to simulate the effect of shear lag (non uniform twist).

- We use a 1st Model is based only on the axial deformability of the section
- We use a 2nd Model considers the deformability of the section by shear, by bending and axial
- And a 3rd Model is a plate model, using the finite element formulation.

The axial rigidity of the wall is maintained. This modelling reproduces a wall element with bar elements. The simple fact of non-consideration of flat sections leads to a decrease in the model's rigidity.

Relationship between the Horizontal Displacements at the top of the shear wall Section obtained by the Theory of Strength of Materials and by the models studied are Flanged Section Shear Wall.

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DISPLACEMENTS IN WALLS DUE TO APPLIED LOADS

Horizontal displacements

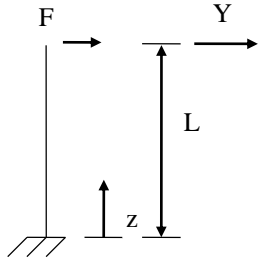


Fig. 1

The horizontal displacements at the core shear center under the action of a concentrated force (F) at the top are given by:

$$z'' = -\frac{M}{EI} \text{ elastic equation}$$

Equation of bending moments: $M = -F(L - z)$

$$y'' = \frac{FL - Fz}{EI} \Rightarrow y' = \frac{FLz}{EI} - \frac{Fz^2}{2EI} \Rightarrow$$

$$y = \frac{FLz^2}{2EI} - \frac{Fz^3}{6EI} = \frac{3FLz^2}{6EI} - \frac{Fz^3}{6EI}$$

$$\delta(z) = \frac{Fz^2}{EI} \frac{3L - z}{6}$$

Missing the 2nd part of the equation, due to distortion

$$\delta(z) = z\gamma = z \frac{\sigma}{G} = \frac{F}{GA} z$$

$$\delta(z) = \frac{Fz^2}{EI} \frac{3L - z}{6} + \frac{F}{GA} z$$

Conclusion of the comparison of the models presented for three types of representative sections:

Analyzed sections:

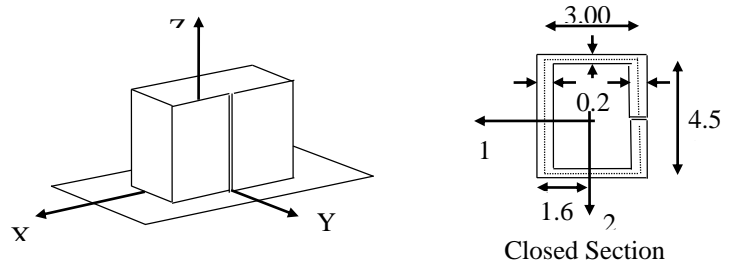
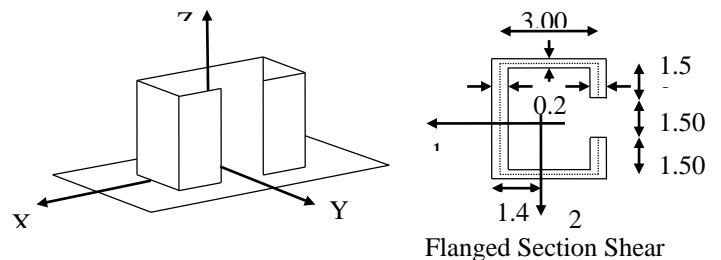


Fig. 2

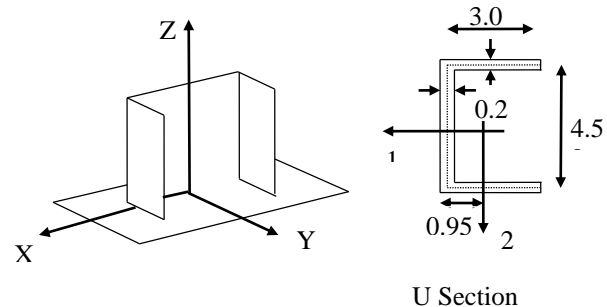
For low shear wall Sections (Figure 2) the importance of shear stiffness is clearly visible. In the one of second model, it is quite evident that, in its formulation, does not enter into its formulation with the cutting rigidity. Hence, in the graph of Figure upper, the large deviation that is observed in relation to the diagram of displacements of the Resistance of Materials in which this effect is felt, is explained. As we saw above in the formulation of the 1st Model, the shear stiffness is not measured. Not considering such an effect makes the model less rigid. This effect is felt mainly at the top floor level.

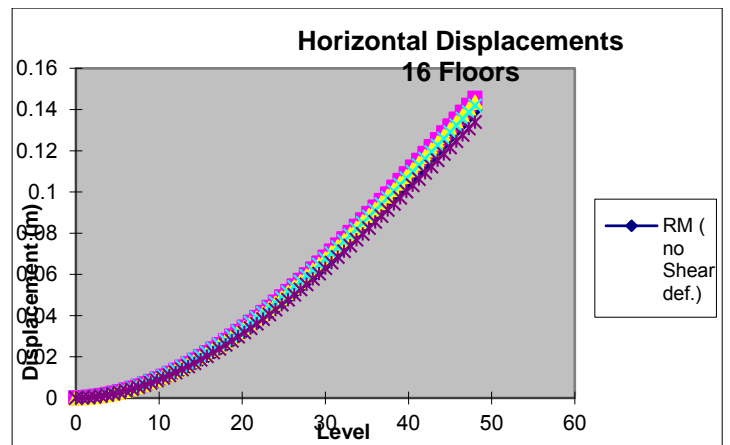
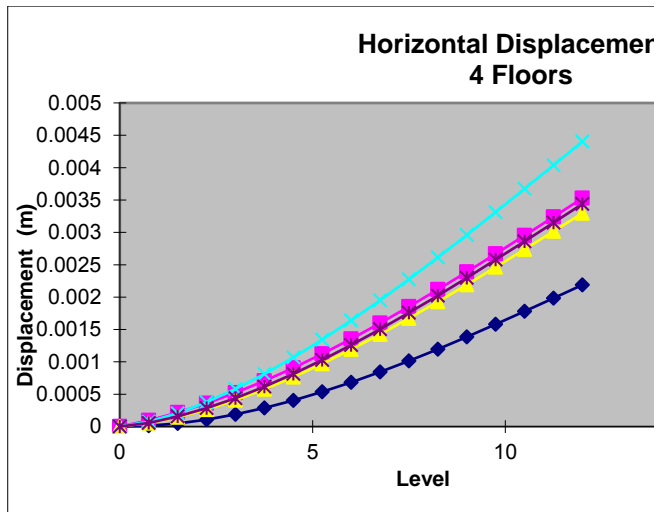
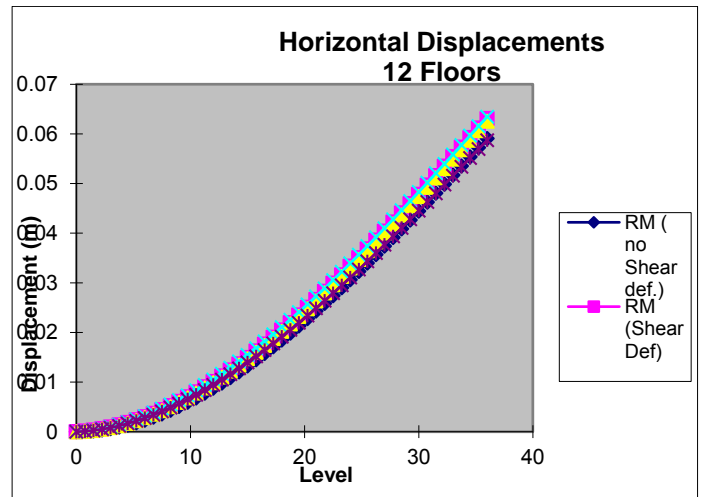
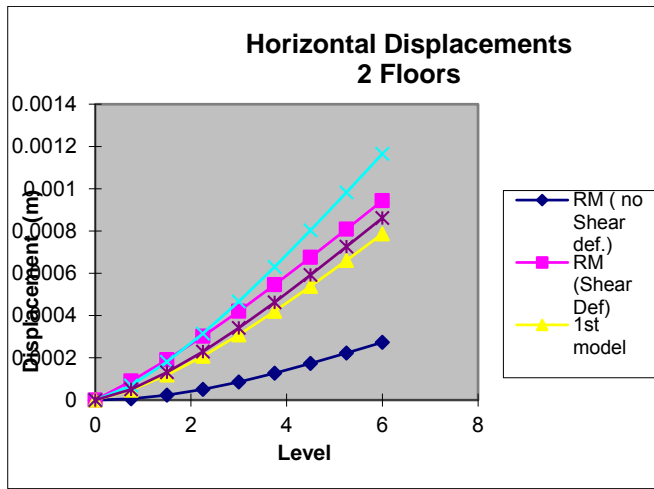
The First Model (only on the axial deformability of the section) and the plate models developed by the University of Berkeley generally present good approximations to the displacements of the Strength of Materials (taking the shear effort in account) although these two models are more rigid (they have smaller displacements):

Relationship between the Horizontal Displacements at the top of the shear wall Section obtained by the Theory of Strength of Materials and by the models studied are Flanged Section Shear Wall. Figure shows Horizontal Displacements

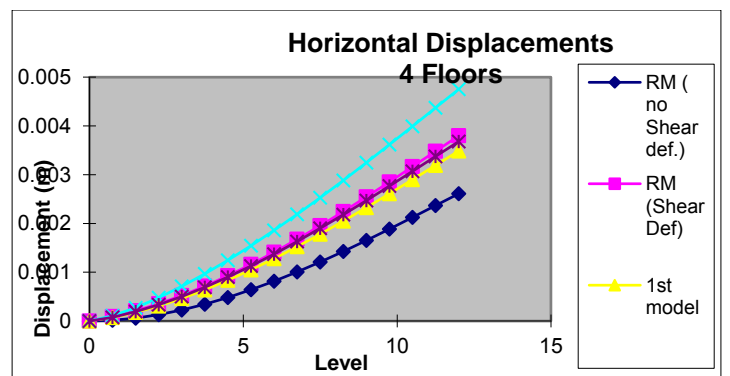
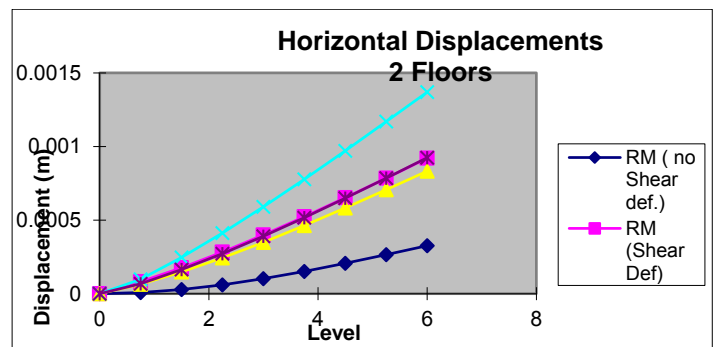
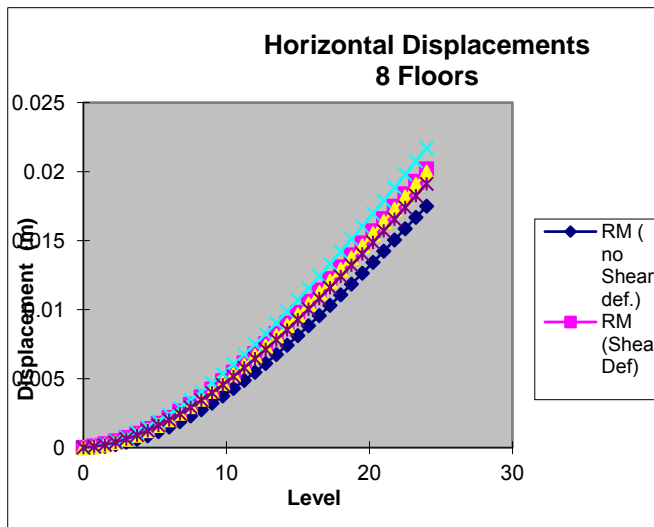


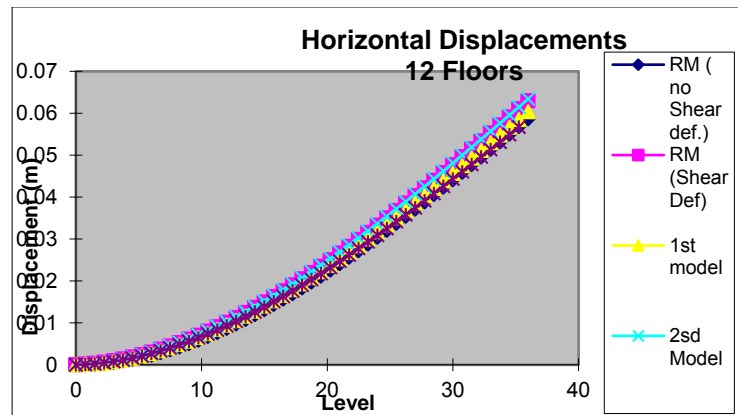
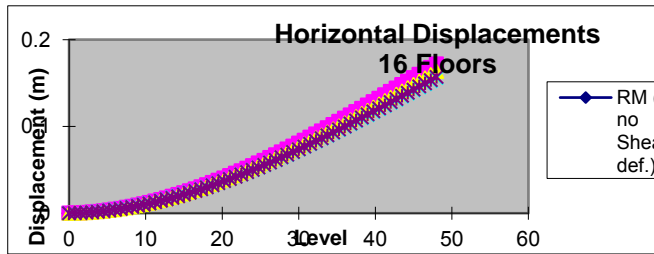
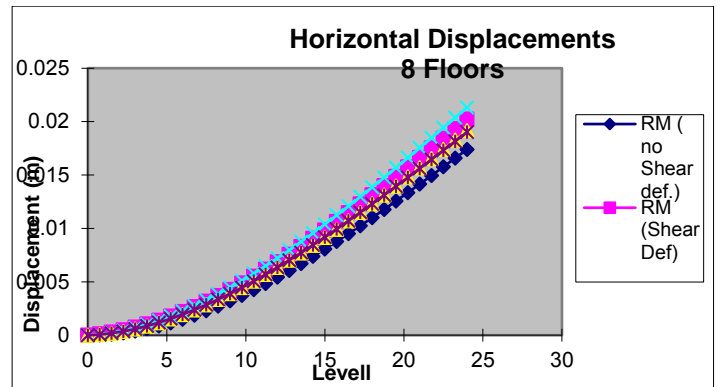
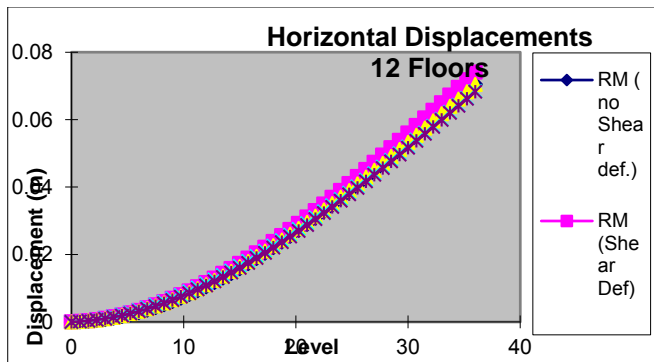
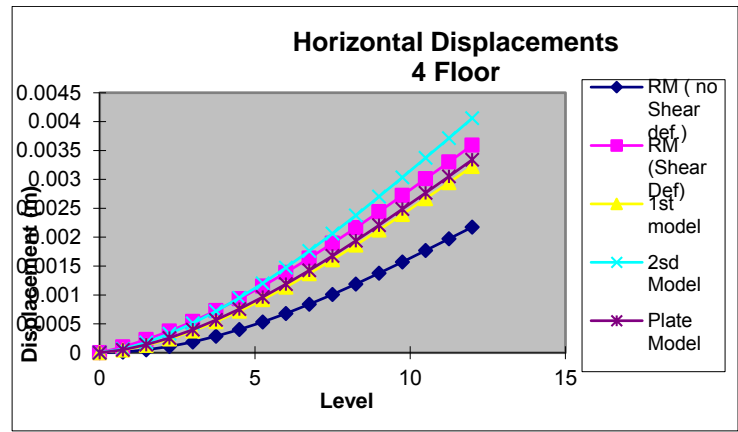
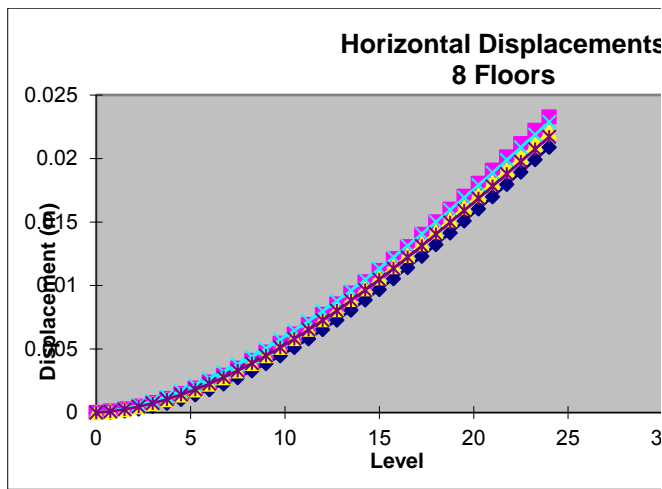
A – Flanged Section Shear Wall



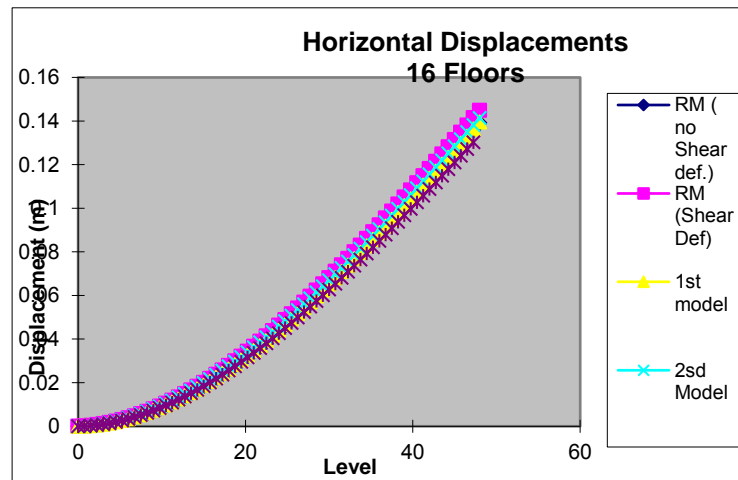
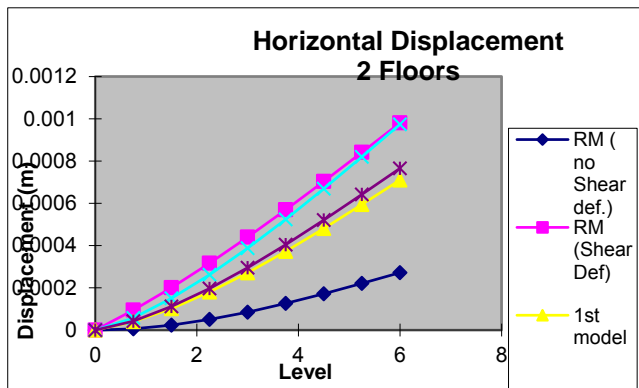


B – U Section





C – Closed Section



| 1stMod | 2 floors (displacements at the top) | | 4 floors (displacements at the top) | | 8 floors (displacements at the top) | |
|--------|---|---|---|---|---|---|
| | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ |
| | 0,238240059 | 0,671982636 | 0,549245067 | 0,79923553 | 0,912896092 | 1,016772909 |
| | 0,234750451 | 0,809769787 | 0,496905338 | 0,801196742 | 0,806292317 | 0,929730225 |
| | 0,382763226 | 1,381149601 | 0,6744421 | 1,114239969 | 0,908128982 | 1,056174749 |

| 1ºMod | 12 floors (displacements at the top) | | 16 floors (displacements at the top) | |
|-------|---|---|---|---|
| | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ |
| | 1,029572812 | 1,081640924 | 1,07128685 | 1,101761805 |
| | 0,930790253 | 0,994122565 | 0,986042483 | 1,023781594 |
| | 0,925627282 | 0,992693232 | 0,984149454 | 1,024259164 |

TABLE I – Table of Horizontal Displacements for the 1st Model

| 2ºMod | 2 floors (displacements at the top) | | 4 floors (displacements at the top) | | 8 floors (displacements at the top) | |
|-------|---|---|---|---|---|---|
| | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ |
| | 0,392105816 | 1,10597815 | 0,748598351 | 1,089325031 | 0,945027524 | 1,052560519 |
| | 0,347811618 | 1,199773371 | 0,663595451 | 1,069963377 | 0,876288856 | 1,010442761 |
| | 0,279227621 | 1,007555302 | 0,535698776 | 0,885023321 | 0,815948715 | 0,948966993 |

| 2ºMod | 12 floors (displacements at the top) | | 16 floors (displacements at the top) | |
|-------|---|---|---|---|
| | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def.}$ $\delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def.}$ $\delta \text{ S.A.P.}$ |
| | 1,000809141 | 1,051422601 | 1,022105036 | 1,051180913 |
| | 0,944224984 | 1,008471414 | 0,975300891 | 1,012628885 |
| | 0,972435293 | 1,0428927 | 0,997238817 | 1,037881994 |

TABLE II – Table of Horizontal Displacements for the 2nd Model

| 3 ^o Mod | 2 floors (displacements at the top) | | 4 floors (displacements at the top) | | 8 floors (displacements at the top) | |
|--------------------|---|---|---|---|---|---|
| | $\lambda = \delta \text{ R.M. N/Shear Def. } \delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def. } \delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def. } \delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def. } \delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def. } \delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def. } \delta \text{ S.A.P.}$ |
| | 0,353311194 | 0,996553396 | 0,709155635 | 1,031929851 | 0,961735193 | 1,071169324 |
| | 0,317340769 | 1,094664426 | 0,636009942 | 1,025485248 | 0,915620088 | 1,05579534 |
| | 0,355062268 | 1,281194424 | 0,650823505 | 1,075219893 | 0,914670873 | 1,063783117 |

TABLE III – Table of Horizontal Displacements for the 3rd Model

| 3 ^o Mod | 12 floors (displacements at the top) | | 16 floors (displacements at the top) | |
|--------------------|---|---|---|---|
| | $\lambda = \delta \text{ R.M. N/Shear Def. } \delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def. } \delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. N/Shear Def. } \delta \text{ S.A.P.}$ | $\lambda = \delta \text{ R.M. Y/Shear Def. } \delta \text{ S.A.P.}$ |
| | 1,032528278 | 1,084745856 | 1,063859862 | 1,09412354 |
| | 1,005867788 | 1,074308483 | 1,045672767 | 1,085694126 |
| | 1,003775748 | 1,076503914 | 1,043770047 | 1,086309636 |

λ - Relation between the horizontal displacements obtained from the Strength of Materials and those obtained by the 3 models under analysis using the SAP200 calculation program.

The parameter λ could be a good indicator between the theory of Strength of Materials, which considers the flat sections after deforming, and the proposed models that essentially intend to show the differences between the sections obtained by the theory of Strength of Materials and the sections that are obtained with each of the formulations presented.

From here it can be concluded when it is important to consider the Shear Lag effect, that is, the fact that we approach or distance ourselves from the flat sections and which are the various effects directly linked.

Almost all models correctly translate the various deformities with the exception of the 1st Mod. For the reasons already described above. Obviously, as we increase in height and the deformability by shear is no longer important, this model begins to approach the others as well as the theory of Strength of Materials itself.

The parameter λ indicates us in the idealized model to simulate the shear wall Section when using a certain type of model, we begin to move away from the linearity of the sections. This factor may be important as it is a good indicator of errors, we are

making regarding the theory that sections remain flat after deforming.

The fact that the parameter λ moving away or approaching 1 indicates the greater or lesser flexibility of the section.

Vertical Displacements

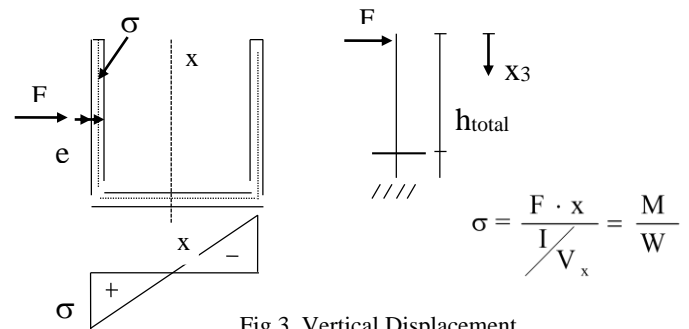
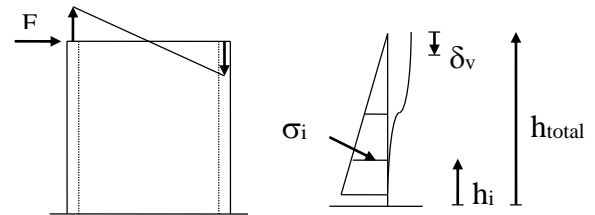


Fig 3. Vertical Displacement



$$\delta_v = \sum \delta_{v_i} = \sum \frac{\sigma_{M_i}}{E} \times h_i \Rightarrow \delta_{v_i} = \frac{\sigma}{E} \times h_i$$

Vertical Displacement on top of console:

$$\delta_v^{topo} = \int_0^L \epsilon dx_3 = -\frac{F v_x}{EI} \int_0^L (L - x_3) dx_3$$

$$\epsilon = \frac{\sigma_M}{E} = \frac{M}{EI / v_x} = \frac{-F(L - x_3)}{EI / v_x} \Rightarrow$$

$$\delta_v^{topo} = -\frac{F v_x}{EI} \times \left[L \times x_3 - \frac{x_3^2}{2} \right]_0^L =$$

$$= -\frac{F v_x}{EI} \times \left[L^2 - \frac{L^2}{2} \right] =$$

$$\delta_v^{topo} = -\frac{F v_x L^2}{EI \cdot 2} \Rightarrow$$

$$\delta_v^{topo} = -\frac{FL^2}{2EI} v_x$$

Parameter indicating the approximation of vertical displacements obtained by the models described to the theory of Resistance of Materials

Parameter indicating the approximation of vertical displacements obtained by the models described to the theory of Resistance of Materials

| | 2 floors (displacements at the top) | | |
|--|--|--|--|
| | $\lambda = \delta \text{ SAP90 (1oMod) - RM } \delta \text{ RM}$ | $\lambda = \delta \text{ SAP90 (2oMod) - RM } \delta \text{ RM}$ | $\lambda = \delta \text{ SAP90 (3oMod) - RM } \delta \text{ RM}$ |
| | 0,278119471 | 0,161154407 | 0,178152746 |
| | 0,421644423 | 0,347348567 | 0,385621204 |
| | 0,412667506 | 0,408695027 | 0,436692248 |

Table 4 – Relationship between areas of vertical displacement diagrams obtained by the models described and by the Strength of Materials for 2 floors




| 4 floors (displacements at the top) | | | |
|---|--|--|--|
| | $\lambda = \frac{\delta_{SAP90(1^{\text{st}}\text{Mod}) - RM}}{\delta_{RM}}$ | $\lambda = \frac{\delta_{SAP90(2^{\text{nd}}\text{Mod}) - RM}}{\delta_{RM}}$ | $\lambda = \frac{\delta_{SAP90(3^{\text{rd}}\text{Mod}) - RM}}{\delta_{RM}}$ |
|  | 0,085438123 | 0,075306201 | 0,079762681 |
|  | 0,208860242 | 0,117119126 | 0,143937157 |
|  | 0,256095202 | 0,167402449 | 0,198502199 |

Table 5 – Relationship between areas of vertical displacement diagrams obtained by the models described and by the Strength of Materials for 4 floors




| 8 floors (displacements at the top) | | | |
|---|--|--|--|
| | $\lambda = \frac{\delta_{SAP90(1^{\text{st}}\text{Mod}) - RM}}{\delta_{RM}}$ | $\lambda = \frac{\delta_{SAP90(2^{\text{nd}}\text{Mod}) - RM}}{\delta_{RM}}$ | $\lambda = \frac{\delta_{SAP90(3^{\text{rd}}\text{Mod}) - RM}}{\delta_{RM}}$ |
|  | 0,12649386 | 0,055577265 | 0,082795592 |
|  | 0,076859581 | 0,045828261 | 0,089067749 |
|  | 0,104111953 | 0,069035691 | 0,104635137 |

Table 6 – Relationship between areas of vertical displacement diagrams obtained by the models described and by the Strength of Materials for 8 floors




| 12 floors (displacements at the top) | | | |
|---|--|--|--|
| | $\lambda = \frac{\delta_{SAP90(1^{\text{st}}\text{Mod}) - RM}}{\delta_{RM}}$ | $\lambda = \frac{\delta_{SAP90(2^{\text{nd}}\text{Mod}) - RM}}{\delta_{RM}}$ | $\lambda = \frac{\delta_{SAP90(3^{\text{rd}}\text{Mod}) - RM}}{\delta_{RM}}$ |
|  | 0,120311169 | 0,05359803 | 0,084576801 |
|  | 0,069969337 | 0,036709153 | 0,086960417 |
|  | 0,081040935 | 0,050432304 | 0,093967488 |

Table 7 – Relationship between areas of vertical displacement diagrams obtained by the models described and by the Strength of Materials for 12 floors




| 16 floors (displacements at the top) | | | |
|---|--|--|--|
| | $\lambda = \frac{\delta_{SAP90(1^{\text{st}}\text{Mod}) - RM}}{\delta_{RM}}$ | $\lambda = \frac{\delta_{SAP90(2^{\text{nd}}\text{Mod}) - RM}}{\delta_{RM}}$ | $\lambda = \frac{\delta_{SAP90(3^{\text{rd}}\text{Mod}) - RM}}{\delta_{RM}}$ |
|  | 0,110617655 | 0,052148998 | 0,088165015 |
|  | 0,067403684 | 0,029273879 | 0,088421207 |
|  | 0,072851125 | 0,046047483 | 0,092225179 |

Table 8 – Relationship between areas of vertical displacement diagrams obtained by the models described and by the Strength of Materials for 16 floors

These values are obtained by direct integration of the area diagrams of displacements for each situation.

This parameter may be indicative of the approximation between the models presented and the theory of Strength of Materials that considers flat sections.

III. CONCLUSION

Shear lagging, resulting from non-uniformity of stress distribution around the connection, is one of the most important design considerations in steel construction since it reduces the load capacity of tension members. Thus, the shear lag impact might be characterized as the non-linear stress distribution (non-uniform or inelastic) resulting in a decreased resistance in tension members. From the analysis of the 3 proposed models, it was concluded that for lower levels and mainly for sections with Flanged Sections Shear Wall, almost forming a rectangle, there is a strong warping (flexural torsional buckling) effect on these walls, with the remaining sections showing a reasonable approximation of the theory of Strength of Materials, although for low levels the Shear Lag effect becomes important although

These values are obtained by direct integration of the area diagrams of displacements for each situation.

This parameter may be indicative of the approximation between the models presented and the theory of Strength of Materials that considers flat sections.

λ - Relation between the horizontal displacements obtained from the Strength of Materials and those obtained by the 3 models under analysis using the SAP2000 calculation program.

The parameter λ could be a good indicator between the theory of Strength of Materials, which considers the flat sections after deforming, and the proposed models that essentially intend to show the differences between the sections obtained by the theory of Strength of Materials and the sections that are obtained with each of the formulations presented.

From here it can be concluded when it is important to consider the Shear Lag effect, that is, the fact that we approach or distance ourselves from the flat sections and which are the various effects directly linked.

Almost all models correctly translate the various deformities with the exception of the 1st Mod. For the reasons already described above. Obviously, as we increase in height and the deformability by shear is no longer important, this model begins to approach the others as well as the theory of Strength of Materials itself.

The parameter λ indicates us in the idealized model to simulate the shear wall Section when using a certain type of model, we begin to move away from the linearity of the sections. This factor may be important as it is a good indicator of errors, we are making regarding the theory that sections remain flat after deforming.

The fact that the parameter λ moving away or approaching 1 indicates the greater or lesser flexibility of the section.

Parameter indicating the approximation of vertical displacements obtained by the models described to the theory of Resistance of Materials

it dissipates. This effect quickly in height. (Roughly at 8 levels this effect is practically no longer felt). Shear-lag effect is a phenomenon typical for the framed tube structures under horizontal loads. The parameter λ defined in two ways clearly indicates that the Shear-Lag is important for low floors. One can analyze the numerous results presented so far both graphically and analytically that we are directly or indirectly led to the same conclusion. The shear-lag effect decreases with the floor levels and then it transfers to negative shear-lag effect which increases up to the top of the building. The shear lag effect decreases when the column spacing increases. The column spacing affects the negative shear-lag effect more clearly at the top half of the building.

REFERENCES

- [1] Moffat, K.R., and Dowling, P.J. Steel box girders, parametric study on the Shear Lag phenomenon in Steel box girder bridges. CESLIC Report BG17. Engineering Structures Laboratories, Imperial College of Science and Technology, London, 1972.

- [2] BIS (1987) IS 875 (Part 3)—1987 Indian standard code of practice for design loads (other than earthquake) for buildings and structures. Bureau of Indian Standard, New Delhi
- [3] Moffat, K.R. and Dowling, P.J. Shear Lag in Steel box girder bridges. *The Structural Engineer*, vol 53, October 1975.
- [4] Coull, A. and Stafford Smith, B., *Tall Building Structures: Analysis and Design*, John Wiley & Sons, New York, 1991. Designbuild-Network
- [5] Bauer, K., and Kristek, V., Stress Distribution In Front Columns of High-Rise Buildings, *Journal of Structural Engineering*, Vol. 119, No. 5, May, 1993. Chicago Architect Foundation
[https://doi.org/10.1061/\(ASCE\)0733-9445\(1993\)119:5\(1464\)](https://doi.org/10.1061/(ASCE)0733-9445(1993)119:5(1464))
- [6] Kwan, A. K. H., Simple Method for Approximate Analysis of Framed Tube Structures, *Journal of Structural Engineering*, Vol. 120, No. 4, April, 1994.
[https://doi.org/10.1061/\(ASCE\)0733-9445\(1994\)120:4\(1221\)](https://doi.org/10.1061/(ASCE)0733-9445(1994)120:4(1221))
- [7] Nagpal, A. K. and Singh, Y., Negative Shear Lag in Framed-Tube Buildings, *Journal of Structural Engineering*, Vol. 120, No. 11, November, 1994.
[https://doi.org/10.1061/\(ASCE\)0733-9445\(1994\)120:11\(3105\)](https://doi.org/10.1061/(ASCE)0733-9445(1994)120:11(3105))
- [8] Kwan, A. K. H., Shear Lag in Shear/Core Walls, *Journal of Structural Engineering*, Vol. 122, No. 9, September, 1996.
[https://doi.org/10.1061/\(ASCE\)0733-9445\(1996\)122:9\(1097\)](https://doi.org/10.1061/(ASCE)0733-9445(1996)122:9(1097))
- [9] Visnjic, G., Nožak, D., Kosel, F. and Kosel, T. (2014), “Reducing shear-lag in thin-walled composite I-beam wing spars”, *Aircraft Engineering and Aerospace Technology*, Vol. 86 No. 2, pp. 89-98.
<https://doi.org/10.1108/AEAT-09-2012-0153>
<https://doi.org/10.1108/AEAT-09-2012-0153>
- [10] Eric Reissner, Least Work Solutions of Shear Lag Problems, 30 Aug 2012, <https://doi.org/10.2514/8.10712>
<https://doi.org/10.2514/8.10712>
- [11] Martin Goland, Shear Lag Solutions for Sheet-Stringer Panels by Means of a Hydrodynamic Analogy, *Journal of the Aerospace Sciences*, 2012
- [12] Giola Santoni-Bottai, Exact Shear-Lag Solution for Guided Waves Tuning with Piezoelectric-Wafer Active Sensors, *AIAA Journal*, 2012
<https://doi.org/10.2514/1.J050667>
- [13] John de S. Coutinho, A Practical Method of Allowance for Shear Lag, *Journal of the Aeronautical Sciences*, 2012
- [14] S. B. Batdorf, Use of shear lag for composite microstress analysis rectangular array, *AIAA Journal*, 2012
- [15] Sisi Xiang, Tuning the deformation mechanisms of boron carbide via silicon doping, *Sci Adv*, 2019
<https://doi.org/10.1126/sciadv.aay0352>
- [16] Xulong QIN, A Study on The Boundary Layer for The Planar Magnetohydrodynamics System, *Selections from Acta Mathematica Scientia*, 2015
- [17] S. B. Batdorf, Note on Shear Interaction between Two Fibers. Technical rept., 1982-09-01, California Univ Los Angeles School of Engineering and Applied Science
- [18] A. K. H. Kwan, Shear Lag in Shear/Core Walls, Univ. of Hong Kong, Pokfulum Rd., Hong Kong. *Journal of Structural Engineering*, Vol. 122, Issue 9 (September 1996), [https://doi.org/10.1061/\(ASCE\)0733-9445\(1996\)122:9\(1097\)](https://doi.org/10.1061/(ASCE)0733-9445(1996)122:9(1097))
[https://doi.org/10.1061/\(ASCE\)0733-9445\(1996\)122:9\(1097\)](https://doi.org/10.1061/(ASCE)0733-9445(1996)122:9(1097))
- [19] Q. Z. Luo, Q. S. Li, J. Tang, Shear Lag in Box Girder Bridges, August 15, 2002, [https://doi.org/10.1061/\(ASCE\)1084-0702\(2002\)7:5\(308\)](https://doi.org/10.1061/(ASCE)1084-0702(2002)7:5(308))
[https://doi.org/10.1061/\(ASCE\)1084-0702\(2002\)7:5\(308\)](https://doi.org/10.1061/(ASCE)1084-0702(2002)7:5(308))
- [20] B.A. Burgan, P.J. Dowling, The treatment of shear lag in design, *Thin-Walled Structures*, Volume 9, Issues 1–4, 1990, Pages 121-134, ISSN 0263-
[https://doi.org/10.1016/0263-8231\(90\)90041-V](https://doi.org/10.1016/0263-8231(90)90041-V)